The Effect of Indefinite Games, Natural Monopolistic Barriers, and Quality Improvements on Socially Optimal Patent Terms

Prepared for

Professor David Gill

Purdue University

Prepared by

Max Lin

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Abstract

This research report investigates the game theory of patent terms, exploring how the exogenous factors of indefinite games, natural monopolistic barriers, and quality improvements influence the optimal term length for social utility. A survey of the current literature landscape was conducted to understand what current theories predict for socially optimal patent terms and what models may be borrowed or extended. With the research gathered, this report applied game theory analysis through numerical derivation to predict how the socially optimal patent terms may vary in relation to the set of exogenous factors and their concerning variables. The results of the report should provide preliminary relationships between each variable and the socially optimal patent term to investigate, as well as highlight noteworthy areas where further research may be warranted to improve patent policy decisions.

Acknowledgement

I would like to thank Professor David Gill for giving me the opportunity to create this report, advising me throughout the initial research phase, and providing feedback on drafts of this report. I would also like to thank John Martinson Honors College at Purdue for providing a pathway which allowed me to consider this report and encouraged me to pursue this report.

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1 Introduction

The writing of this report was motived primarily by a personal interest in understanding the factors that affect the patent term which best benefits society. The general understanding of patents as tools to encourage disclosure of innovation and investment into said innovation at the expense of temporary monopolies is foundational (Hall & Harhoff, 2012), however this broad interpretation leaves no indication for how this tradeoff should be balanced. Industry innovators and policy makers would benefit from understanding how relevant industry and economic environment variables affect the socially optimal patent term, so this report set out to investigate a set of possible relationships.

1.1 Methodology

Results from this report are entirely theoretical, and the mathematics used to derive certain results rely on numerical derivation as opposed to algebraic. The derived utility functions became inseparable when analyzing the first order condition, thus making algebraic solutions difficult. Lacking the mathematical tools to find a solution with this method, a numerical method was employed instead. This method optimizes over two independent variables by graphing over one variable as the independent variable and setting the second variable as an adjustable constant. Using a graphing utility, the optimal value of the first variable can be found through inspection. The second variable is optimized by varying the adjustable constant and observing how the dependent variable changes. While these models may be somewhat limited by this method of derivation, the insights gained from this report should highlight potential areas for further research.

The background information, the explanation for Nordhaus' theory of optimal patent length, and the ideas for the indefinite game rely on research conducted by others, with sources listed in the References section. All the analysis conducted in the Results section were completed without external references. Where information from other sources is present, source authors were credited within the text.

1.2 Significance of the Report

The purpose of this report is to investigate the potential influence of considering an indefinite game, natural monopolistic barriers, and quality improvements on the socially optimal patent term. The results from the modeling and analysis provide preliminary relationships between these factors and the socially optimal patent term, valid to the accuracy of numerical analysis. Noteworthy relationships serve as points of interest and areas where future studies may benefit from exploring to construct more accurate patent models compared to those that do not consider such exogenous factors.

1.3 Intellectual Property Fundamentals

To provide context behind the significance of this report, a solid background in the functions of intellectual property is necessary. Intellectual property (IP) stems from intellectual objects, or information, which have a certain value that may benefit from protection and restriction. In the modern knowledge-based economy, IP has become the bedrock of social organization and production. Digitalization and globalization have made information invaluable, and so protection for IP has become increasingly important. Whole organizations may rely on IP laws to maintain competitive advantages, and even entire industries surround the creation and transaction of intellectual objects.

IP is protected primarily through three instruments: copyrights, patents, and trade secrets. Copyrights generally apply to written and creative media, giving owners the exclusive rights to reproduce, prepare derivative works from, distribute copies of, and publicly perform or display their work (Hettinger, 1989). The protection provided is often relatively narrow in scope but long in duration. Patents give their owners the exclusive right to make, use, and sell an invention regardless of how anyone else comes up with it (Hettinger). These protections tend to be broader but limited in duration. Trade secrets, unlike the previous two protections, can apply to nearly any intellectual object and may be kept indefinitely, however it only prevents improper acquisition of the IP. This report exclusively investigates patents, however further generalizations could be made in future studies to apply to any form of IP protection.

2 Models

The following section identified different levels of assumptions and generated utility functions for players based on these assumptions. While important equations and graphics were embedded in this section, the complete proofs can be found in the Appendix. Analysis of the results of these games were conducted in the Results section.

2.1 Relevant Metrics and Relationships

When considering the agents in a patent game, four distinctive classes of agents come to mind: consumers, governments, inventors, and firms. Typically, consumers and governments are represented as the broader class of society as governments are theoretically representative of consumers. This report followed the conventional practice of capturing these two agents in the broader class of society. The nuance between consumers and governments is that society's utility is based on the utility of consumers while the strategy chosen by society is done so through governments. Governments should ideally represent their consumers, but differences in the motivations and utilities of the two agent classes may be possible avenues of future research.

The ultimate goal of IP protection is to increase social welfare, so analyzing the utility of general society is important for advising on designing patent systems and policy solutions. The split between inventors and firms is a subtle distinction that highlights the nuance between the individual who produces the patents and the institutions which enable and use them (Hettinger, 1989). The more complex utility function for individual inventors could be investigated further to yield a more effective patent system compared to profit-seeking firms, however this report took the traditional approach of capturing inventors within firms to simplify models.

One of the advantages of limiting the set of players to firms and society is that the utility function for a profit-maxing entity is relatively straightforward, represented by the real profits it would obtain for a given strategy profile. For this report, all units of profit were denoted in terms of the USD ($). The social benefits of patents are complex but were modeled with simple welfare analysis by examining the consumer surplus gained as a result of the patents. This method of analysis frames social utility in common units with firm utility, allowing for unambiguous comparisons of the results of patents on firms and society, at the cost of the fidelity of the models.

Patents themselves have varying qualities which help enforce IP protection, such as patent term, scope of claims, screening severity, price of fees, and so on. All of these contribute to the broader strength of patents in a patent system; however, this report investigated the relationship between the patent term length and their effects on social welfare. Investigation into other patent strength variables may yield a more optimal patent system.

The foundation of every model in this report relies on games with the following properties:

1) $N = \{s, f_1, f_2, \dots, f_n\}$ where s represents society, f represents a firm, and $n \geq 1$ represents the number of firms.
2) $S_s = T = [0, \infty)$ where T represents the patent term in years set by the patent system chosen by society.
3) $S_i = I = [0, \infty)$ where I represents the lump-sum R&D investment in USD firm i chooses to put towards a patent.
4) Society always moves first.
5) Firms move simultaneously for each innovation discovery stage.

The utility functions of the players vary depending on the model used and factors considered, but these rules hold for all models observed.

2.2 Nordhaus' Theory of Optimal Patent Life

To formally describe the utility function for firms and society, much of the economics were borrowed from Nordhaus' theory of optimal patent life through Scherer's geometric reinterpretation (1972). A generic competitive market for a normal good was used as the market of analysis (Figure 2.2.1) with the generic firm having an initial constant marginal cost C_0 and a pre-patent market price P_0 equal to C_0, producing at competitive equilibrium quantity Q_0. Patents have the effect of providing a cost reduction B in terms of percentage of C_0, and are isolated in this model to only one firm and one innovation discovery stage. The value of B for a patent is a function of a firm's initial lump-sum R&D investment I, with a decreasing marginal utility and hence a downward concavity, along with the obvious upper asymptote at 100%. The models in this report used a typical exponential decay function

$$B(I) = 1 - e^{-\alpha I} \tag{2.2.1}$$

where α is a constant representing innovation difficulty, but more advanced and accurate models such as a logarithmic growth function could be explored so long as these functions have a long-term downward concavity and an upper asymptote at 100%.

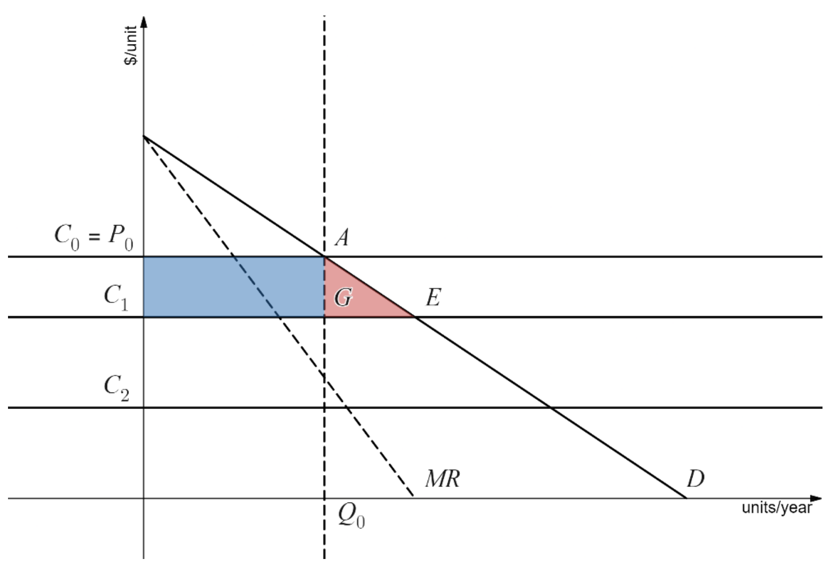

Figure 2.2.1. Demand and cost curves for a generic firm in a competitive normal good market.

With the cost reduction $B(I)$ from a patent, the firm then shifts its marginal cost to C_1 and earns a normal rent of C_0AGC_1 per year from profit or licensing the technology to recover the equivalent profit. A firm cannot charge a price higher than P_0 as the market is still competitive and the patent is assumed to not change the quality of the good, so full monopoly rents cannot be realized. Should B be sufficiently large, however, marginal cost may reach a point such as C_2 where the monopoly price is lower than P_0 and monopoly quantity greater than Q_0, resulting in a region of diminishing returns from investment. For this report, only patent innovations that do not allow for full monopoly operation, or "run-of-the-mill" innovations as termed by (Scherer, 1972) were considered.

The real rent R can be found as a function of B and T using the continuous discounted cashflow (DCF) analysis to yield the relationship

$$R(B,T) = BQ_0P_0\left(\frac{1-e^{-\lambda T}}{\lambda}\right) \tag{2.2.2}$$

where $\lambda = ln(1+r)$ and r is the discount rate (derivation in the Appendix). The utility function of a generic firm i can be generalized as $u_i = R - I$, but a more useful version utility function would be in terms of the two strategy variables each player controls. By substitution, the utility function of a generic firm i based on Nordhaus' theory of optimal patent life becomes

$$u_i(I,T) = Q_0P_0(1-e^{-\alpha I})\left(\frac{1-e^{-\lambda T}}{\lambda}\right) - I \tag{2.2.3}$$

The trade-off that Nordhaus identified is captured by the consumer surplus AEG that society must postpone until the patent expires and competition drives the equilibrium price down to C_1. The utility function of society can similarly be calculated as a function of I and T using continuous DCF to create equation (2.2.4) where γ is the elasticity of demand (derivation in the Appendix).

$$u_s(I,T) = \gamma(1 - e^{-\alpha I})^2 \frac{e^{-\lambda T}}{\lambda} \tag{2.2.4}$$

2.3 Indefinite Games

An indefinite game relaxes certain assumptions imposed by Nordhaus' theory and considers competition between firms at each stage. Each firm was assumed to have identical utility functions for simplicity and demonstration purposes. One of the more important differences to highlight, besides the multiple stages of innovation discovery that were modeled, is the probabilistic nature of innovation discovery captured in this model. While Denicolò (2000) used a Poisson discovery process to model innovation, a simpler probabilistic factor described in equation (2.3.1) was used in this report where I_θ represents the aggregate R&D investment by other firms. Combined with equation (2.2.3), the utility function for a firm where there are multiple competing firms can be represented with equation (2.3.2).

$$\sigma_i(I_i, I_\theta) = \frac{I_i}{I_i + I_\theta} \tag{2.3.1}$$

$$u_i(I_i, I_\theta, T) = Q_0 P_0 \left(\frac{I_i}{I_i + I_\theta}\right)(1 - e^{-\alpha I_i})\left(\frac{1 - e^{-\lambda T}}{\lambda}\right) - I_i \tag{2.3.2}$$

Social utility would also be slightly altered as the consumer surplus would depend on the investment of the firm that obtains the patent. Capturing all firms under a summation, the utility function for society in a probabilistic stage would result in equation (2.3.3) where I' represents the aggregate investment by all firms and I represents the set of investment strategies by all firms.

$$u_s(I,T) = \frac{\gamma e^{-\lambda T}}{\lambda I'} \sum_{i=1}^{n} I_i (1 - e^{-\alpha I_i})^2 \tag{2.3.3}$$

2.4 Considering Natural Monopolistic Barriers

One factor not considered by previous models, and rarely analyzed in present literature, is that of natural monopolistic barriers. Most patent models assume the competition instantly adopt the patented technology after the patent expires and at no cost, which is realistically not the case as there tends to be some lag time for competitors to adopt the patented technology and cost to perfect it. Both effects were shown to increase the utility of the lead firm and decrease the utility of society.

Adoption lag can be considered an extra time of $T^{+} < T$ which the lead firm enjoys its monopoly. While other firms may choose to start development earlier to utilize it at the time the patent expires and drive T^{+} to 0, a firm risks the loss of investment should the original patent be extended or risk fees for accusations of using unlicensed patented technology. This adoption lag thus increases the utility of the lead firm by allowing its monopoly to reign for longer and decreases the utility of society as it needs to wait longer for the realization of consumer surplus.

Developing new technology is not free, but neither is adoption. Competition that adopts the patented technology would incur an imitation cost of $I^{+} = aI_L$, where I_L is the investment by the lead firm for this technology and a is an arbitrary coefficient less than 1 representing the proportion of imitation cost relative to development cost. This would tend to drive the equilibrium price up for a period of time so other firms can recuperate this cost. During this time, the lead firm would also enjoy rents equivalent to that imitation cost, while society would suffer from a small portion of its consumer surplus not being realized for a longer period. The imitation cost of would be recovered from after the patent expires, so the lead firm would gain an extra $\frac{aI}{(1+r)^T}$, transforming the utility of a firm from equation (2.3.2) to equation (2.4.1).

$$u_i(I_i, I_\theta, T) = Q_0 P_0 \left(\frac{I_i}{I_i + I_\theta}\right)(1 - e^{-\alpha I_i})\left(\frac{1 - e^{-\lambda T}}{\lambda}\right) + \left(\frac{I_i}{I_i + I_\theta}\right)\left(\frac{aI_i}{(1+r)^T}\right) - I_i \qquad (2.4.1)$$

Social utility is proportional to the square of the difference between marginal cost and market price, multiplied by the demand elasticity. This is because the triangle of consumer surplus has its height dependent on price difference and the base dependent on the height multiplied by the slope of its hypotenuse, or demand elasticity. That price difference can be assumed to be linear with respect to I^+, and over the period this price difference lasts, it can be reasonably assumed I^+ is fully felt by society, adjusted for future discount. This equates to society losing a utility of $\frac{\gamma(aI)^2}{(1+r)^T}$

$$u_s(I,T) = \frac{\gamma e^{-\lambda T}}{\lambda I'} \sum_{i=1}^{n} [I_i(1 - e^{-\alpha I_i})^2] - \frac{\gamma(aI)^2}{(1+r)^T} \qquad (2.4.2)$$

2.5 Considering Quality Improvements

All previous models and much of existing literature address patents as intellectual objects that reduce the cost of producing a certain good, however patents may also be useful to firms if they increase the quality of the good. Rather than a cost reduction percentage B, a patent may produce a marginal utility increase percentage β instead. The goods produced by the firm owning this patent see higher demand, and since it can differentiate its product from other non-patented products, the firm can command a monopoly and aim for monopoly pricing. The pricing power is limited to the same β increase, however, as the ratio of marginal utility to price must not exceed that of the unpatented goods or consumers would not demand the patented good. Assuming the increase in demand and price still yield the equilibrium quantity Q_0, the rent earned by the firm would be P_1AGP_0 (Figure 2.5.1). The cost to society would be the delay of realizing consumer surplus AEG.

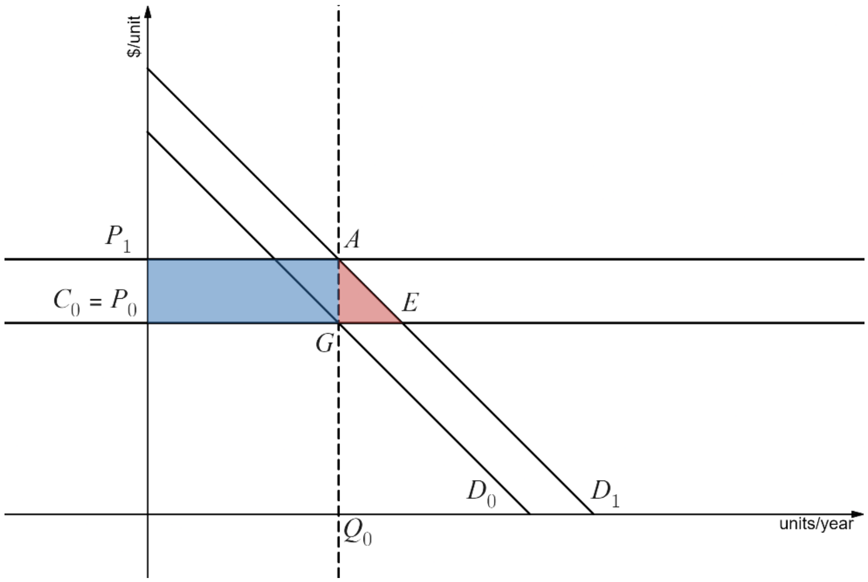

Figure 2.5.1. Demand and cost curves for a firm with a quality-improving patent in a competitive normal good market.

Since β does not necessarily have an asymptote, the natural log function was used to relate it with I. The new utility function for firms can then be derived to be equation (2.5.2), and the new utility function for society can similarly be found to be equation (2.5.3).

$$\beta(I) = ln(\alpha I + 1) \quad (2.5.1)$$

$$u_i(I_i, I_\theta, T) = Q_0 P_0 \left(\frac{I_i}{I_i + I_\theta}\right)(\ln(\alpha I_i + 1))\left(\frac{1 - e^{-\lambda T}}{\lambda}\right) + \left(\frac{I_i}{I_i + I_\theta}\right)\left(\frac{aI_i}{(1+r)^T}\right) - I_i \quad (2.5.2)$$

$$u_s(I, T) = \frac{\gamma e^{-\lambda T}}{\lambda I'} \sum_{i=1}^{n} [I_i (ln(\alpha I + 1))^2] - \frac{\gamma (aI)^2}{(1+r)^T} \quad (2.5.3)$$

3 Results

The analysis of the models proposed in the previous section was conducted numerically using a graphing utility to gather a basic understanding of how adding different exogenous factors affect the optimal patent term. While these results may be limited in applicability, they provide possible starting points for future studies regarding the variables mentioned in this report.

Important figures and equations are included in this section; however, more complete derivations can be found in the Appendix.

3.1 Game Theory Analysis

The optimal patent term T can be found at the sub-game perfect Nash equilibrium (SPNE) of each model. Firms maximize their utility given any T and the investment of other firms, and society takes this knowledge to optimize its own strategy to yield SPNE T^*. Using the arbitrary baseline values of $r = 12\%$, $Q_0 = 1.5$, $P_0 = 1.5$, $\alpha = 0.5$, and $\gamma = 2$, T^* can be found numerically by varying T for equations (2.2.3) and (2.2.4) on a graph between I and utility (Figure 3.1.1). The baseline patent term calculated with this model returns $T^* = 5$ years.

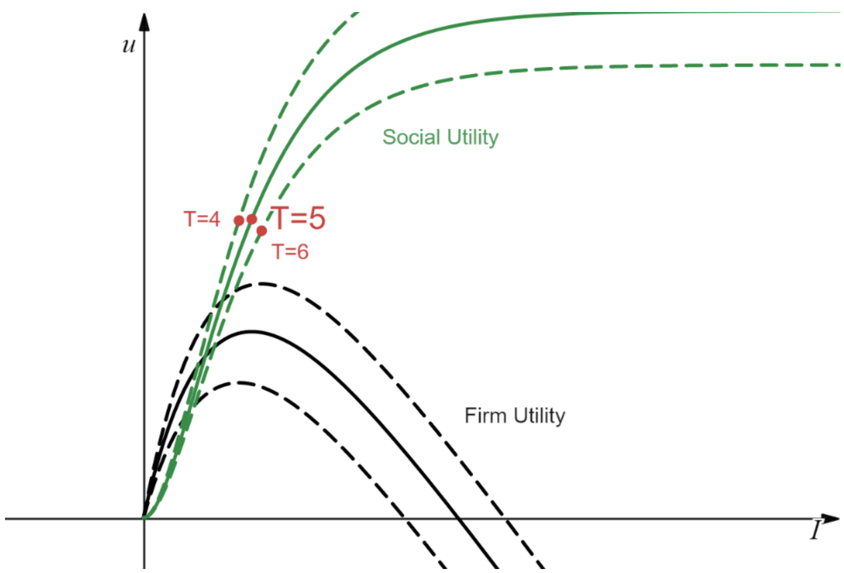

Figure 3.1.1. Numerical optimization for T^* with isolated firms in a patent game.

When considering interactions between firms across multiple stages of innovation discovery, the patent game becomes indefinite, and each innovation discovery stage can no longer be isolated. To demonstrate this, an indefinite game between two firms was considered.

Given that the firms are identical, the equilibrium investment strategy for both firms must also be identical. This reduces the equilibrium utility for both firms to equation (3.1.1). Likewise, the utility for society reduces from equation (2.3.3) back to (2.2.4) where I represents the investment strategy for any firm since they are identical. Absent of a cooperative strategy, the SPNE must be where the optimal point of a firm utility coincides with the equilibrium utility as no firm would want to deviate their strategy given the other's response (Figure 3.1.2). The optimal equilibrium point, however, yields higher utility for both firms while drastically reducing social utility, achievable through legitimate cooperation strategies.

$$u_i(I_i, T) = \frac{Q_0 P_0}{2}(1 - e^{-\alpha I_i})\left(\frac{1-e^{-\lambda T}}{\lambda}\right) - I_i \tag{3.1.1}$$

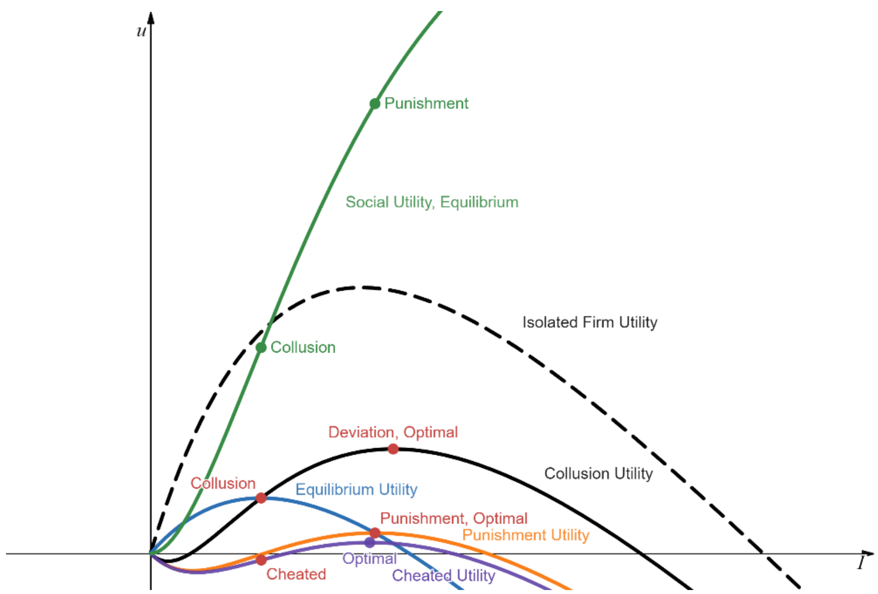

Figure 3.1.2. Firm and social utility for an indefinite game with grim trigger strategies.

One such cooperation strategy would be for both firms to employ a grim trigger strategy, with payoffs captured in table 3.1.1. Incorporating the baseline values, two firms colluding would yield a payoff higher than if they both played at Nash equilibrium. Given the collusion level of investment by one firm, however, the other firm would yield a higher payoff if they deviated to their optimal investment level. A failure of cooperation would then lead to both firms investing at the Nash equilibrium level, representing the punishment case.

Table 3.1.1. Payoff matrix of two firms playing grim trigger strategies in an indefinite patent game.

Firm 1 \ Firm 2	**Collude**	**Deviate \ Punish**
Collude	(0.766, 0.766)	(-0.091, 1.443)
Deviate \ Punish	(1.443, -0.091)	(0.25, 0.25)

Figure 3.1.2 illustrates that society would clearly prefer both firms to operate at Nash equilibrium as this would yield a significantly higher social utility. The likelihood of sustaining cooperation between firms, however, depends on the typical innovation time τ and the discount rate r as this determines how much firms discount future returns from future innovation discovery stages. Combined with net present value analysis, the discount coefficient δ can be found as a function of τ and r in equation (3.1.2). For the particular game described in table 3.1.1, the δ per stage would have to be over 0.567 to sustain collusion (derivation in the Appendix).

$$\delta = \frac{1}{(1+r)^{\tau}} \tag{3.1.2}$$

A small r and/or a small τ would mean a higher tendency to collude, so society would prefer values of r and τ just large enough to break down collusion without negatively impacting social utility, since δ also impacts how society discounts future utility. This is not something society has direct control over, as it depends on the industry (τ) and the economic environment (r). Conducting another numerical optimization over the cases of collusion and punishment using baseline values, the optimal patent term under collusion T_c^* comes out to be 7 years (Figure 3.1.3) while the optimal patent term under punishment T_p^* remains at 5 years, although social utility at 6 years is closer to optimal whereas 4 years is closer for isolated firms (Figure 3.1.4). These values suggest that when firms are able to collude with each other under an indefinite patent game, patent terms should be longer, while a lesser lengthening may be needed, if at all, when firms cannot collude.

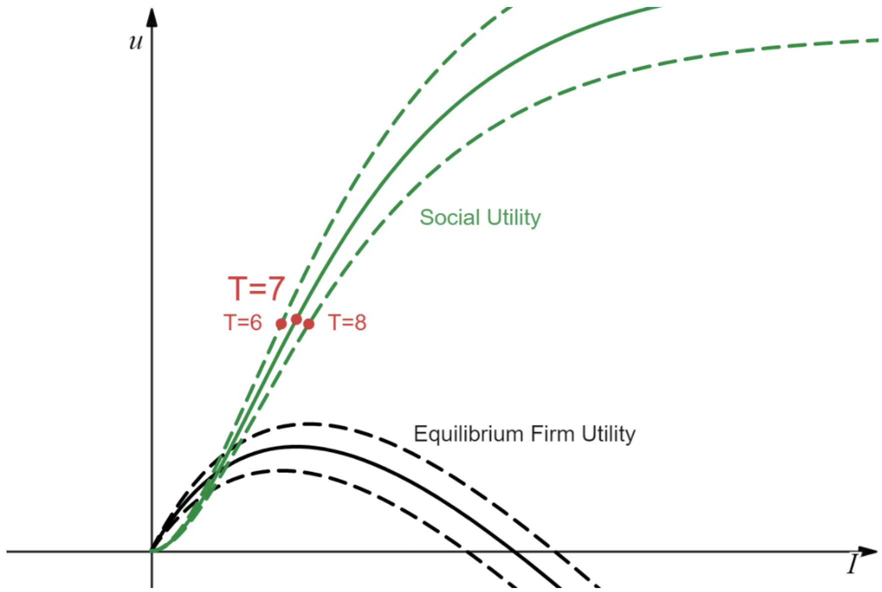

Figure 3.1.3. Numerical optimization for T^* with firms colluding in an indefinite patent game.

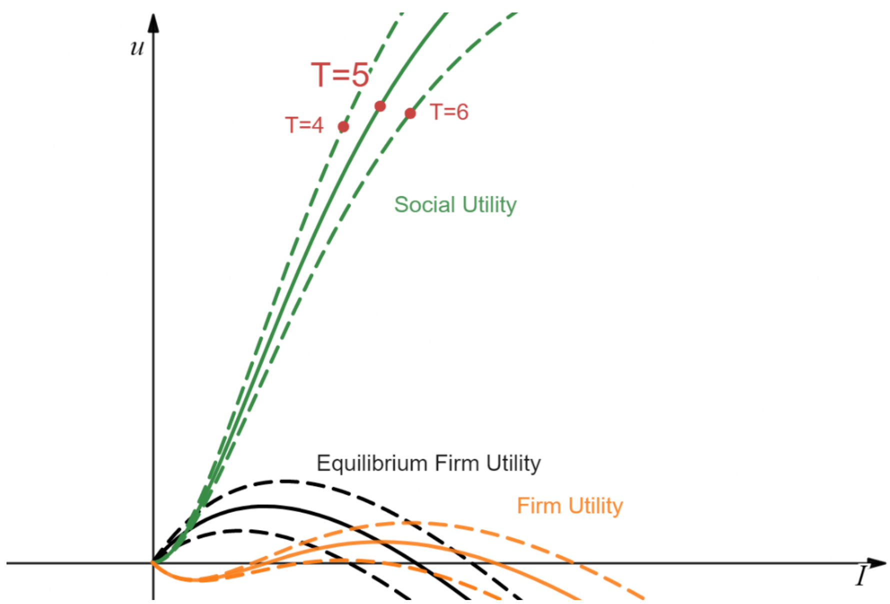

Figure 3.1.4. Numerical optimization for T^* with firms failing to collude in an indefinite patent game.

Extending the model for *n* number of firms, while the absolute values of investment, utility, and patent term may vary, it holds that collusion between firms will always be less preferred for society, and that society will need a longer patent term to capture the optimal social utility when firms collude.

Once monopolistic barriers are introduced, T^* becomes pushed down as these factors extend the lead firm's monopoly to a certain extent beyond models without considering these barriers. The adoption lag of T^+ can quite straight-forwardly be accounted for by lowering T^* to equal its original value, implying a strong relationship that suggests a lower T^* the higher T^+ is for an industry. Identifying T^+ may not be trivial, however, and may also not be constant even within an industry or a firm, so dynamic patent term systems may be an area of exploration appropriate to address this variable. Imitation cost I^+, when graphing equations (2.4.1) and (2.4.2) in place of equations (2.3.2) and (2.3.3) for either collusion or Nash equilibrium, yielded very little difference compared to figures 3.1.3 and 3.1.4 although firm utilities do increase slightly and social utility dips a small amount. From the preliminary analysis, it appeared that imitation cost has little effect on T^*.

If a patent were to increase the quality of a product rather than decrease the cost to produce it, T^* likely changes as well. Plotting for the collusion case using equations (2.5.2) and (2.5.3) and baseline conditions, T^* comes out to be 9 years, a significant increase over the 7 years when the patent produces a cost reduction instead (Figure 3.1.5).

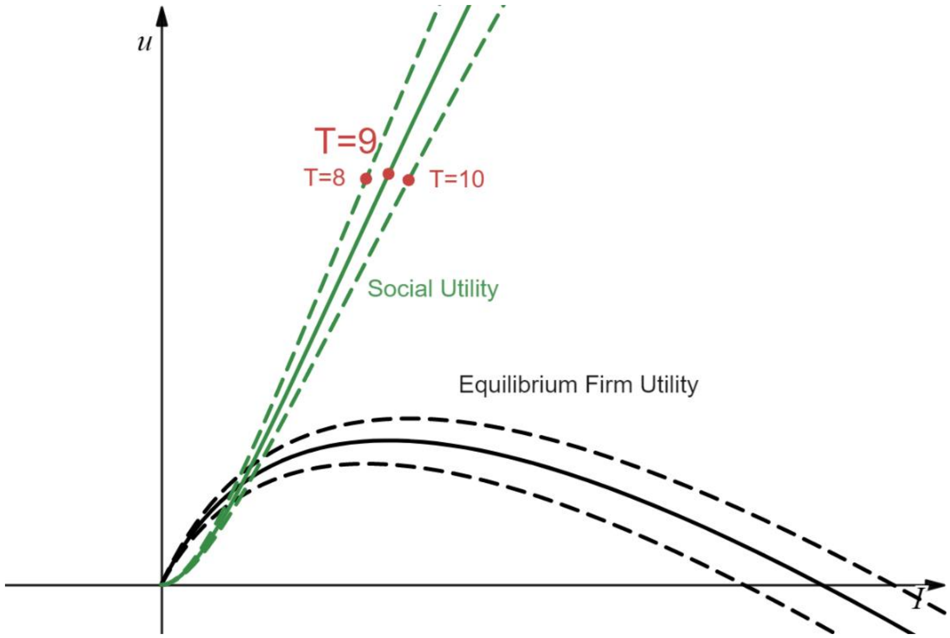

Figure 3.1.5. Numerical optimization for T^* with firms colluding in an indefinite patent game and with quality improvements.

One explanation for this increase in T^* could be due to the unbounded nature of quality improvements compared to cost reductions. β may exceed 100% while B may not, and the resulting utility was likely amplified due to the quadratic relationship with price difference in social utility. The assumption of using the same α variable may not have translated well, however the degree of change in T^* still indicates that patents which increase product quality may require a longer term to be socially optimal.

3.2 Conclusion

The general conclusion this report has found is that the investigated factors produce significant variations in the socially optimal patent term. Starting from Nordhaus' theory of optimal patent life as the base model, extending the game to be indefinite yields two potential equilibria dependent on the discount rate and the typical innovation time. When firms collude, the utility of firms are greater at the expense of smaller social utility. Under collusion, society needs a longer patent term to capture the optimal social utility. Society benefits more when firms are unable to collude, sometimes more than the base case with a small number of firms. Natural monopolistic barriers tend to push the optimal patent term lower, with the effect from adoption lag being relatively significant and the effect from imitation cost being barely significant. Should patents increase the quality of a good instead of lowering the cost to produce it, the socially optimal patent term could be higher. The net effect of the combined factors depends on various industry and economic environment variables, but the generalized direction of these effects should remain consistent.

3.3 Recommendations

The results of this report were able to establish preliminary relationships for the investigated factors, however these results should not be applied to formal policy decision making as they are limited in accuracy and mathematical robustness. Studies which can take these initial findings and provide a proper mathematical derivation may yield results which would prove more useful in practice. Empirical studies investigating the outlined relationships would help confirm or refute the theories presented in this report. Extending the investigation of exogenous factors may also strengthen any theoretical models derived in the future for analyzing patent systems.

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Appendix

A.1 Derivation of equation (2.2.2)

i) Continuous DCF equation: $\int_{T_1}^{T_2} F(t)e^{-\lambda t}dt$ where $F(t)$ is the cash flow rate

ii) $F(t) = BQ_0P_0$, a constant that can be taken out of the integral

iii) For the period of interest, $T_1 = 0$ and $T_2 = T$

iv) $\int_0^T e^{-\lambda t}dt = \frac{1-e^{-\lambda T}}{\lambda}$

v) $R(B,T) = \int_0^T BQ_0P_0e^{-\lambda t}dt = BQ_0P_0\left(\frac{1-e^{-\lambda T}}{\lambda}\right)$

A.2 Derivation of equation (2.2.4)

i) AG is equal to *B*, and knowing γ, GE is equal to γB

ii) Continuous DCF equation: $\int_{T_1}^{T_2} F(t)e^{-\lambda t}dt$ where $F(t)$ is the cash flow rate

iii) $F(t) = \gamma B^2$, a constant that can be taken out of the integral

iv) For the period of interest, $T_1 = T$ and $T_2 = \infty$

v) $\int_T^\infty e^{-\lambda t}dt = \frac{e^{-\lambda T}}{\lambda}$

vi) $u_s(I,T) = \int_T^\infty \gamma(1-e^{-\alpha I})^2e^{-\lambda t}\,dt = \gamma(1-e^{-\alpha I})^2\frac{e^{-\lambda T}}{\lambda}$

A.3 Derivation of δ for section 3.1

i) The utility for colluding is $0.766\sum_{i=0}^{\infty}\delta^i = \frac{0.766}{1-\delta}$

ii) The utility for deviating is $1.443 + 0.25\delta\sum_{i=0}^{\infty}\delta^i = 1.443 + \frac{0.25\delta}{1-\delta}$

iii) To sustain collusion, $\frac{0.766}{1-\delta} > 1.443 + \frac{0.25\delta}{1-\delta}$

iv) Separation of variables results in $1.193\delta > 0.677$ or $\delta > 0.567$